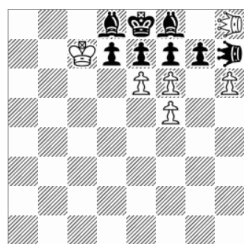## Entry No. 1:

Author: Walter Lindenthal
Publication: Schwalbe (issue 305, p 693, #18364, dedicated Thomas Brand), October 2020

last move? (6+8)
make&take
b) wPf5→h5

 Mulehopper (MU)

a) R wMUf8-h8(bBg8→f8)+
b) R wMUg6-f8×bSh8(bBg8→f8)+

The wMU must have arrived on h8 as the "last move" in order to set up check against bK. In twin a) the only black piece that could have lastly made a black move is the bMU, which could have moved bMUf5-h7(wPg6→f5), but as a "last move" per se this would have been illegal due to the still undefended check against the bK! In twin b) there is no black piece at all that could have moved lastly. So, how did the checking wMU get to h8 in a "last move"? The wMU either moved there without capture (like in a)) or with capture of a black Knight under make&take (like in b)). In both cases he kicked the bB from g8 to f8, which looks in the diagram deceptively like an unmoved bBf8, due to the closed Pawn-rank 7 in front of it.

In twin a) the last black move anyhow can only have been bMUf5-h7(wPg6→f5), but then the only legal solution would allow a wMU already having stayed on f8, waiting there for its "last move" to h8 (because g6(h7) would not have been available as a starting point for the wMU then). In twin b) the bMU can only have stayed on h7 already before, so the last black move must have been bKf8-e8 ("protected" from check by wMUg6 because of the make-part in the make&take condition), at the same time clearing f8 for the wMU's "last move" capturing the bSh8 under make&take.

Mulehopper: Moves and captures like a Grasshopper, in addition kicks (like a Mule) the hurdle back to the Mulehopper's starting square of the move.

Entry No. 2:

Author: Walter Lindenthal
Publication: Schwalbe (issue 308, p 89, #18556), April 2021

-2(w+b) & h#1 (15+10)
help retractor

 Oppohopper-Grasshopper (OG)

R 1.Pb3×Ra4 Rd4-a4 2.OGb6-e3 Pd7×Se6 & forward 1.Rg8-f8 Se6-g7#

First you have to find a black hurdle for the wOG to retract to b6, so that the latter finally can cover one of the black King's available flight squares on d8. Uncapturing the bRa4 which retracts to d4 will solve the problem, at the same time later a bRd4 cannot intervene in the forward play (such as an uncaptured bQd4 instead could do on g7; or even a possibly promoted bBd4 could do after its assumed but mistimed [see Pawn promotions!] uncapture R 1.Pb2×Bc3...)! The bPe6 has to uncapture (wSe6) in order to close the Pawn-rank 7 and as such taking another one of the black King's available flight squares (a non-closure of the Pawn-rank 7 would not result in a sound solution). The final forward move of the bRg8-f8 takes the black King's last available flight square and at the same time goes against the need to subsequently capture Sg7 in defence of its check to bK. As the uncaptured wSe6 (as the only available white stone for uncapturing by bPe6!) could not serve (by capturing bPc7) as a hurdle for the wOGb6 on c7 (because of same colour!), the wSe6's only sensible move is (non-capturing) to g7#.

In Retro play considering all legally possible (and at the same time purposeful) uncapturing options, the positions of the (especially white) Pawns in the diagram / pawnstructure, the number (and assumed location) of captured stones in black (and white), and especially legality in potential Pawn promotions (file switchovers, "local" availability of opponent's stones, etc) being contingent on some interdependent other factors as aforesaid, the necessary soundness of the unique legal solution requires the given magnitude of stones (15+10) on the board. The main influencing factors are:

White Stones: 7 wP in the diagram, 1 wP promoted before to wOG;
15 white stones in the diagram, 1 wS missing from definite capture bPd7×wSe6; **=16**
--> no further legal uncapture of white stones in Retro play!
--> no further file switchover by black Pawns can have occured (--> promotion squares?)!

Black Stones: 6 bP in the diagram, all 6 exactly being allocated; 2 bP yet undefined incident;
10 black stones in the diagram, at least 3 pieces(!) captured by wP on files e,f,h;
--> leaves 3 stones unaccounted for (2 b Pawns/promotions?, 1 b piece/bRa4?); **= 16**
--> bPg7 could not have left his file g! Was it captured by a white piece(!) or did it promote to what(?) stone on g1 (=black field)?
--> Thus, only 2 black stones were available for "unknotting" the promotion issue of the white promotion to wOG on the left side files...

Oppohopper-Grasshopper: A Grasshopper who must not use a hurdle of his own color.

Entry No. 3:

Author: Walter Lindenthal
Publication: Schwalbe (issue 310, p 210, #18684), August 2021

last move? (4+11)
Kamikazecirce
b) wPg7→b6

 Mulehopper (MU)

a) R wMUd7×Sb7(Kc7→d7) [+bSg8, -wMUb7/b8]
b) R wMUd7×Sg7(Kf7→d7) [+bSb8, -wMUg7/g8]

As the bK is under a double check threat, the "last move" can only have come from a white stone! The double check threat is only possible if either the move of the white stone opened both lines of attack at the same time (e.g. wSe6×bSc7, but in the diagram this would have led to a rebirth of wS on g1?!, conversly on b1?!), or the white move pushed the bK into the focus of both attack lines on d7. Here a non-capturing move is impossible, because that would have left the respective white stone on the board, but neither wRa7 nor wBf5 could have made such a move! Under Kamikazecirce both stones in a capture are reborn (type Circe and Paracirce, legal even though rebirth square is occupied!), a capturing fairy stone like the MU is reborn on its own colour promotion square of the file of capture. 7 bP are on the board, the 8th bP was used for promotion to bMU. So, no further black Pawn promotion could have taken place, nor could there have been a further black fairy stone. Consequently, in case a bS was actually reborn in the solution, his current appearance in the diagram proves the rebirth square must have been empty before the rebirth (there cannot be an additional promoted bS)!

What other rebirth cases could have taken place: The rebirth of a black fairy stone (actually impossible!) after a capture on file d would not leave any hints in the diagram (square d1 is occupied by wK!), no further rebirth of a black orthodox stone (other than both bS) can have taken place (empty rebirth squares on rank 8 else), and neither no rebirth of a white orthodox stone is possible (except a here inappropriate wQ, empty rebirth squares on rank 1 else).

Thus, the only solution requires a push of the bK to d7 by a white fairy-stone like MU, which captures on b7 (pushing the bKc7→d7, like in twin a)) or on g7 (pushing the bKf7→d7, like in twin b)), correspondingly capturing a bS on b7 (white square, rebirth of bS on empty g8, occupied rebirth square b8 for MU) or a bS on g7 (black square, rebirth of bS on empty b8, occupied rebirth square g8 for MU). Consequently the particular "second" bS in each twin solution stood already before on b8 (in twin a)), respectively on g8 (in twin c)) in this mirror-inverted setup. Both twins lead to the "disappearance" of the capturing stone MU in the diagram (due to a particularly occupied rebirth square), and the rebirth of the captured bS on their respective, empty rebirth squares. All kinds of capturing on different squares and with different stones being involved in the capture would not have led to the given diagram! In twin b) the bK could not have been placed before on c7 (like in twin a)), because of the additional (triple) check threat by wPb6; and in twin a) the respective square g7 is even occupied by wPg7.
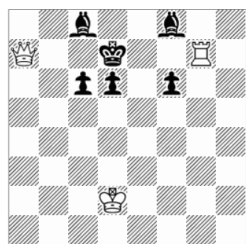

Mulehopper: Moves and captures like a Grasshopper, in addition kicks (like a Mule) the hurdle back to the Mulehopper's starting square of the move.

Kamikazecirce: as Circe, in addition the capturing unit is also reborn (type Paracirce).

Entry No. 4:

Author: Walter Lindenthal
Publication: feenschach (issue 244, p 64, #12444), February-April 2021

last move?   (3+6)
make&take
Strict Circe, ParaSymmetrycirce
b) wKd2→g2

a) R wBc7-d8×Be7 [+bBf8, -wBe7/d2]
b) R wSe7-d8×Bb7 [+bBc8, -wSb7/g2]

As the bK is under a double check threat, the "last move" can only have come from a white stone! The double check threat is only possible if either the move of the white stone opened both lines of attack at the same time (here orthodox conditions alone could not do this), or the white move was a capturing move under make&take condition that acted on both sides left and right of the bK. Strict Circe requires an empty rebirth square for the captured stone (else the capture is illegal), whereas ParaSymmetrycirce follows Paracirce rules for the capturing stone (does not require an empty rebirth square to be a legal capture). The latter rebirth uses a rebirth square mirrored symmetrically from the capture square. Thus, a rebirth of the capturing stone does not happen on his square of origin, but as a function of the capturing square. If the resulting rebirth square is occupied, the capturing stone disappears!

Only such white stones can be involved in the capture (left or right of the bK on rank 7) that do not threaten the bK under make&take(!) with check even before their "last move", only wBc7 (here c8/c6/(d6) not empty!) or wSe7 (here f8/f6 not empty!) fulfill this criterion. Contrariwise, illegal would be wBe7/f7 or wSb7/c7, or any wR or wQ. The introductory make-part of a capture move under make&take must be non-capturing to an empty(!) square as a starting square for the take-part of the move. Also any wP disqualifies due to the specific rules of make&take for wP on rank 8 (see Laue in f-235).

As proved by the empty rebirth squares on rank 8 only a bB can have been captured under make&take and been reborn under Strict Circe rules. The capture must have taken place on e7 (for twin a)) or on b7 (for twin b)), as only these capture squares result in the mirrored but occupied (by wK) rebirth squares, else the capturing stone would still have to be seen in the diagram! Any other captured stone or capture square resulting in a rebirth square that is empty is disapproved by the given diagram, like e.g. wBc7-e8/e6×Sf7, or wBb7-d8×Se7, or even an exchange of both solutions...

Taking all of that into consideration, and the fact that the capturing stone evidently "disappeared" from the board, the summary logic for the twin solutions is:
a) wKd2 → capture square e7 (black square) → wB×bB → +bB(free f8) / -wB (occupied d2)
      (the make-part here must have moved exactly to d8, because d6 is occupied)
b) wKg2 → capture square b7 (white square) → wS×bB → +bB(free c8) / -wS (occupied g2)
Consequently the particular "second" bB in each twin solution stood already before on c8 (in twin a)), respectively on f8 (in twin c)) in this mirror-inverted setup.
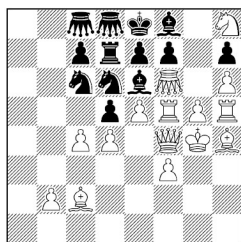
Strict Circe (= Strong Circe): as Circe, but the captured unit "must" be reborn (requires a free rebirth square), otherwise the capture is illegal.

ParaSymmetrycirce: as ParaCirce, but the capturing unit is reborn symmetrically with respect to the centre of the board. As with ParaCirce generally the capture is legal even if the rebirth square is already occupied, thus it's Rex Exclusive (Kings must not capture)!

Entry No. 5:

Author: Walter Lindenthal
Publication: feenschach (issue 246/Internet, p 180, #12591), July–September 2021

-2(w+b) & h#1 (15+13)
help retractor

 Grasshopper (G)

R 1.Sg6×Rh8 Rg8-h8 2.Sh8×Pg6 g7-g6 & forward 1.Rg8×Sh8 Gf6×Rh8#

The solution emerges in the North-Eastern corner through a repetition of sacrifices (especially bR and wS), and retraction to the (partial, right-side) original position of the orthodox stones on the board (especially closed Pawn rank 7, and what kind of stones can legally be placed behind it). The Retro play creates in preparation of the forward play the exact framework for the final, undefendable mate move of the wG (hurdle, black stone for capture / R sacrifice, closing the escape route for the bB), all under special consideration of those legal limitations.

In rectracting correctly to the position at the start of the forward play, all white and black Pawns will be accounted for, this will prove no further promotions could have taken place. All Pawns in the diagram can have reached their position without any capture. The Pawn promotion to the wG must have taken place on b8 (wP[a2]×Qb~ before bP[a7] had reached this rank, considering bP[b7] leaves file b timely to file a or c by capturing the bS), and Pawn promotions to both bG either sequentially on a1 (bP[a7], bP[b7]×Sa~ before wP[a2] had passed this rank on file b), or on a1 (bP[a7]) and on c1 (bP[b7]×Sc~ after wP[c2] had passed this rank). Thus, all 2x16 stones are exactly accounted for, and this way the magnitude of stones results in a unique and sound solution for the given stipulation!