WCCI 2019 – 2021 Section G (Fairies)

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All six problems of this package are chess-mathematical problems of a special kind.

Все шесть задач этой посылки – шахматно-математические задачи особого вида.

Each of the 6 presented chess problems is the result of solving an inverse problem.
A detailed presentation of the theory of composing problems of this type can be found in my article "Inverse Problems in Chess Composition" (http://chess-kopyl.com.ua/ua/ 12/31/20).

Statement of the inverse problem in general form:
It is necessary to compose a chess-mathematical problem for a seriesmover mate (stalemate), which has N solutions, where N is an element of some given subset W of the set of natural numbers.

In particular, a given subset of natural numbers can be represented by a single number, a set of numbers, it can be defined by a number series or described by an attribute property of its elements (curly numbers, palindromes, repunits, repdigits, Smith numbers, Fibonacci numbers, Catalan numbers, significant dates, and etc.).

In the direct problem it is necessary to find:
1) at least one solution of the problem (basis);
2) the number of problem solutions.

The multiplicity of solutions arises due to possible permutations of moves, but without branching and duals. In the solution, each piece moves strictly along its own trajectory from the initial position to the final one. Any solution can serve as a basis. It is convenient to choose a basis in which all trajectories of the solution are sequentially presented.

When counting the number of solutions N of the direct problem, methods of combinatorics and methods of calculations on lattices are used.

Let the solution of some problem Ser.h#n-N? contains two trajectories of length **m** and **k** moves, respectively. In this case, **n = m+k**. The number of solutions to be found. We build a lattice containing m*k cells and (m+1)*(k+1) nodes. (In this case, we get a two-dimensional lattice. In the general case, the lattice will be multidimensional – in terms of the number of trajectories of the solution.) A move along one trajectory will be displayed on the lattice by moving one vertical step upwards, and a move along another trajectory by moving one step horizontally to the right.

The lattice constructed in this way will reflect the nature of the interaction of trajectories. The lattice can be interpreted as the phase space of solutions to the problem. Lattice nodes – points in phase space – correspond to solution positions from the initial position (lower left corner) to the final position (upper right corner).

Each solution of the problem is a trajectory of the phase point from the initial position to the final one. Each cell records the number of ways to reach the top right node of the given cell from the starting position. The number in the top right cell of the grid is called *the sum of the lattice* (S); it is equal to the number trajectories of phase point connecting the initial and final nodes of lattice.

This premise presents three types of problems:
1. **Centuria:** a problem like Ser.h#n-N? There is a functional relationship between the number of moves and the number of solutions: N = n*101 (n <100). W is a set of numbers of the form N = n*101.
2. **Millennium:** a problem like Ser.h#n-N?, where the number of solutions N can be represented as N = d*1001. In this case, W is a set of numbers of the form N = d*1001, (1 ≤ d ≤ 999).
3. **Palindrome:** a problem like Ser.h#n-N? (or Ser.h=n-N?), in which the number of solutions N is a palindrome. In this case, W is the set of numerical palindromes.

In one problem, a combination of signs of two or more types is possible (for example, task No. 6).

The above theory developed for the series-helpmate genre remains valid for the series-selfmate (problem No.3) and, in particular, for the series-reflexmate (problem No.4).

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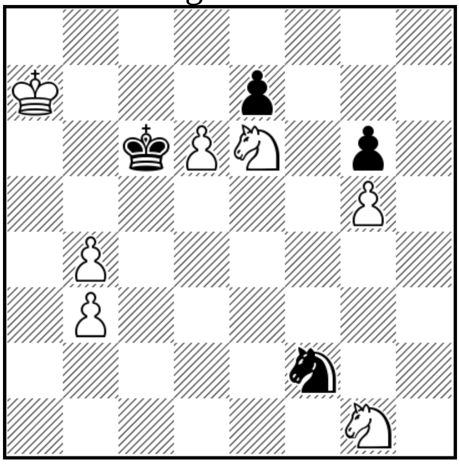
ChessblogYG 13.01.20



Ser.h #20-N? 7+4

How many different solutions?

8/K3p3/2kPN1p1/6P1/1P6/1P6/5n2/6N1

Publication:
ChessblogYG
http://9148.od.ua/stati-i-zametki/
Э.Эйлазян «Обратная задача в шахматной композиции» Часть 1. №3.
13.01.20.

Сайт В. Копыла
http://chess-kopyl.com.ua/ua/
Эдуард Эйлазян Обратные задачи в шахматной композиции. №4.
31.12.20.

Chess-mathematical problem on series-helpmate
Find the (basic) solution and the number of solutions N.

Solution №1.
1.Nd3 2.Nxb4 3.exd6 4.d5 5.d4 6.d3 7.d2 8.d1R 9.Rxg1 10.Rxg5 11.Rc5 12.g5 13.g4 14.g3 15.g2 16.g1B 17.Kb5 18.Bh2 19.Bc7 20.Ba5 Nd4#.
Number of solutions. 1st method. Combinatorics.
1. The move Kb5 is autonomous. It can be done on any move (20 possible ways);
2. Of the remaining 19 moves, 2 moves are made by the knight and 17 moves by pawns (before and after promotion);
3. The trajectories of the knight and the e7-pawn intersect on the d3-square (obstruction);
4. Before obstruction 2 moves of the knight and 3 moves of the pawn can be executed in C^2_{2+3} ways.
5. After obstruction 2 moves of the knight and the remaining 12 moves can be done with C^2_{2+12} ways;
6. Therefore, 19 moves Black can make $C^2_5 + C^2_{14} = 10 + 91 = 101$ way;
7. Taking into account the move of the king, we get the total number of solutions **N = 101 * 20 = 2020.**

2nd method. Construction of a lattice for interconnected trajectories.

3	6	10	10	11	13	16	20	25	31	38	46	55	65	76	88	101
2	3	4		1	2	3	4	5	6	7	8	9	10	11	12	13

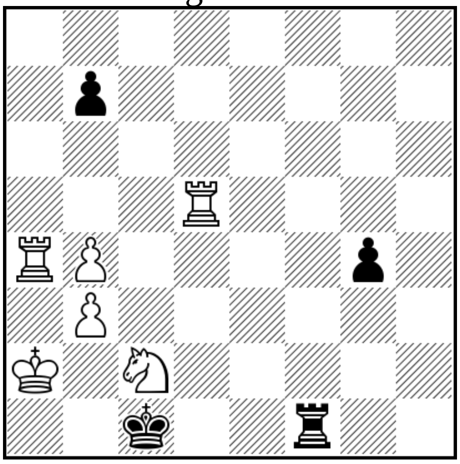
Lattice for the problem Ser.h #20-N?

The sum of the lattice S=101. Taking into account the autonomous trajectory, N=S*20=101*20=2020.

Content of the problem:
1. The number of solutions to the problem is N = 2020, and the number of moves is n = 20. Centuria.
2. Ideal mate in the center of the board.
3. Excelsior, two underpromotions; three active blockings with different pieces.

№2. Eduard Eilazyan

ChessblogYG 18.02.20



Ser.h=18-N? 6+4

How many different solutions?

8/1p6/8/3R4/RP4p1/1P6/K1N5/2k2r2

Publication:
ChessblogYG
http://9148.od.ua/stati-i-zametki/
Э.Эйлазян «Обратная задача в шахматной композиции» Часть 2. №6.
18.02.20.

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Эдуард Эйлазян Обратные задачи в шахматной композиции. №6.
31.12.20

Chess-mathematical problem on series-helpstalemates
Find the (basic) solution and the number of solutions N.

Solution №2.
1.g3 2.g2 3.g1B 4.Bd4 5.Kd2 6.Kd3 7.Ke4 8.Be5 9.Kf5 10.Ke6 11.Bd6 12.Kd7 13.Kc6 14.Bc5 15.Kb5 16.Rf4 17.Rxb4 18.b6 Nxb4 stalemate.

Number of solutions. We build a lattice for interconnected trajectories.
The solution contains 3 trajectories. The **b6** move is autonomous. It can be done on any move (18 possible ways). The long trajectory consists of the moves of the bishop-pawn and the king – 15 moves. The short trajectory consists of two rook moves: Rf4 and Rxb4. The two paths are related by overlapping the second move of the short trajectory. Overlap zone – 5 moves.
For two connected trajectories, we construct a 2x15 lattice. The starting position corresponds to the lower left node of the lattice. Moves by short trajectory– vertical shift upwards, moves by long trajectory – horizontal shift to the right. The numbers in the cells correspond to the number of ways to reach the top right node of the cell.

3	6	10	10	10	10	10	10	20	31	43	56	70	85	101
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

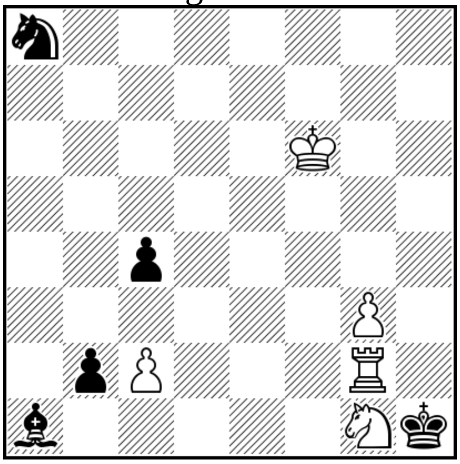
The lattice of the problem Ser.h=18-N?

The sum of the lattice S=101. Taking into account the autonomous trajectory N=18*S=18*101=1818.
The number of solutions is **N = 1818.**

Content of the problem:
1. The number of solutions to the problem N=1818, and the number of moves is n=18. Centuria.
2. Far march of the black king to the stalemate square.
3. Concerted actions of the bishop and the king. Multiple self-pinning and self-unpinning in similar situations with similar motivations.

№3. Eduard Eilazyan

ChessblogYG 26.12.20



Ser.s#20–N? 5+5

How many different solutions?

n7/8/5K2/8/2p5/6P1/1pP3R1/b5Nk

Publication:
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Э.Эйлазян «Обратная задача в шахматной композиции» Часть 3. №19.
26.12.20.

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Эдуард Эйлазян Обратные задачи в шахматной композиции. №19.
31.12.20

Chess-mathematical problem on series-selfmate
Find the (basic) solution and the number of solutions N.

Solution №3.
1.g4 2.g5 3.g6 4.g7 5.g8B 6.Bxc4 7.Bg8 8.c4 9.c5 10.c6 11.c7 12.c8B 13.Bb7 14.Bxa8 15.Be4 16.Bh7 17.Kg718.Kh8 19.Nh3 20.Rxb2 zz Bxb2#.

Number of solutions. We build a lattice for interconnected trajectories.
The solution consists of three trajectories: one autonomous (Nh3) and two interconnected (2 moves of the white king and the remaining 17 moves; obstruction on the square g7).
The configuration of the trajectories of the solution coincides with the configuration of the trajectories of problem No.1. Therefore, the problems have identical lattices of interconnected trajectories.

3	6	10	10	11	13	16	20	25	31	38	46	55	65	76	88	101
2	3	4		1	2	3	4	5	6	7	8	9	10	11	12	13

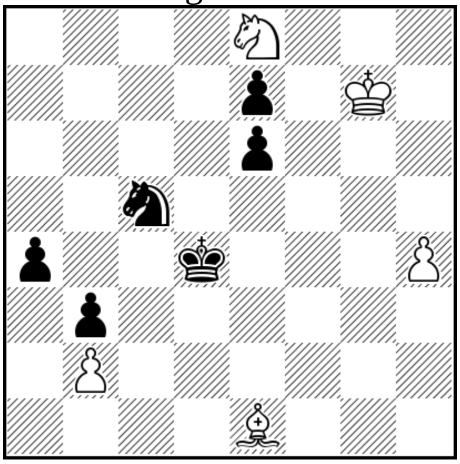
Lattice of problem Ser.s#20-N?

The number of solutions considering the autonomous trajectory N=S*20=101*20=2020. **N = 2020.**

Content of the problem:
1. The number of solutions to the problem N=2020, and the number of moves is n=20. Centuria.
2. Final zugzwang
3. Excelsior, two underpromotions of white pawns into light-squared bishops, two active blockings.

№4. Eduard Eilazyan

ChessblogYG 26.12.20



Ser.r#20-N? 5+6

How many different solutions?

4N3/4p1K1/4p3/2n5/p2k3P/1p6/1P6/4B3

Publication:
ChessblogYG
http://9148.od.ua/stati-i-zametki/
Э.Эйлазян «Обратная задача в шахматной композиции» Часть 3. №20.
26.12.20.

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31.12.20.

Chess-mathematical problem on series-reflexmate
Find the (basic) solution and the number of solutions N.

Solution №4.
1.h5 2.h6 3.h7 4.h8R 5.Rh3 6.Rxb3 7.Rf3 8.Bh4 9.b3
[or 9.Bxe7 10.b3 11.bxa4 12.a5 13.a6 14.a7 15.a8B!
Try 15.a8Q? 16.Qc8 17.Qxe6 18.Qg8 19.Rf7 20.Kf8 Ne6#. But 18.Bf6#! reflexmate to the Black!
16.Bd5 17.Bxe6 18.Bg8 19.Rf7 20.Kf8 Ne6#]
10.bxa4 11.a5 12.a6 13.a7 14.a8B!
Try 14.a8Q? 15-16.Qxe6 17.Qg8 18.Bxe7 19.Rf7 20.Kf8 Ne6#. But 17.Bf2#! reflexmate to the Black!
15.Bd5 16.Bxe6 17.Bg8 18.Rf7 19.B:e7 20.Kf8 Ne6# reflexmate to the White.
The solution consists of three trajectories: one autonomous (Kf8) and two interconnected (two of the bishop's moves and the remaining 17 moves; blocking on **h4** and overlapping on the same square).

We find the number of solutions by the lattice.

	3	6	10	11	13	16	20	25	31	38	46	55	65	76	88	101
1	2	3	4	1	2	3	4	5	6	7	8	9	10	11	12	13

Lattice for problem Ser.r#20-N?

The sum of the lattice S=101. Taking into account the autonomous trajectory, we get the number of solutions N=20*101=2020. **N = 2020.**

Content of the problem:
1. The number of solutions to the problem is N=2020, and the number of moves is n=20. Centuria.
2.Two tries with a reflexmate to the black king.
3. Model mate with the last piece.
4. Excelsior, two underpromotions, three active blockings.

№5. Eduard Eilazyan

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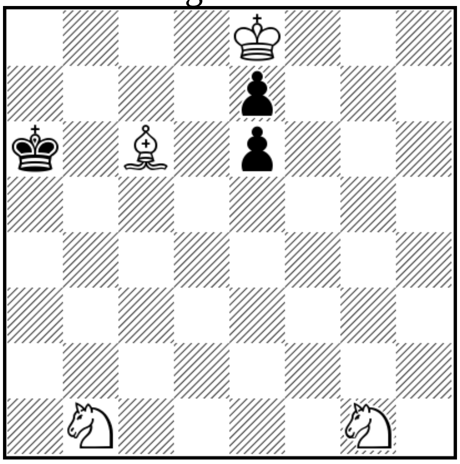

Ser.h#17–N? 4+3

How many different solutions?

4K3/4p3/k1B1p3/8/8/8/8/1N4N1

Publication:
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http://9148.od.ua/stati-i-zametki/
Э.Эйлазян «Обратная задача в шахматной композиции» Часть 3. №28.
26.12.20.

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Эдуард Эйлазян Обратные задачи в шахматной композиции. №28.
31.12.20.

Chess-mathematical problem on series-helpmate
Find the (basic) solution and the number of solutions N.

Solution №5.

1.Ka5 2.Kb4 3.Kb3 4.Kc2 5.Kd1 6.e5 7.e4 8.e3 9.e2 10.e1B 11.Bd2 12.Bc1 13.e5 14.e4 15.e3 16.e2 17.e1B Ba4# – beautiful model epaulette mate.

To find the number of solutions, we need a little more theory.

Catalan's scheme. Let us consider such configurations in which the interacting trajectories partially overlap, i.e. the initial part of the first trajectory runs along the same squares as the final part of the second trajectory.

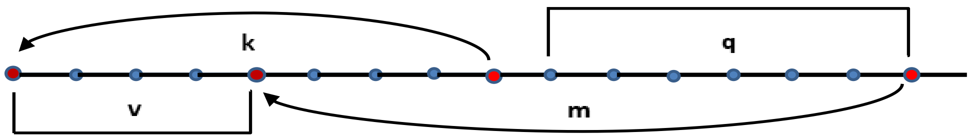


Рис.1. Схема Каталана

Catalan's scheme C(k, m, q) has three independent parameters: **k** is the length of the first trajectory, **m** is the length of the second trajectory, and **q** is the length of the “free” part of the second trajectory. Restrictions are imposed on the parameters: $q < m \le k + q$; $v = k+q+1-m$. For each set of values of the parameters of the Catalan scheme, you can build a lattice and calculate its sum. In the general case, the sum is calculated by binomial coefficients: $S = C^{m}_{m+k} - C^{m-q-1}_{m+k}$. If the length of the autonomous trajectory is **a** moves, then the number of moves in the solution of the problem is **n = a+k+m**, and the number of solutions is calculated by the formula $N = S * C^{a}_{n} = (C^{m}_{m+k} - C^{m-q-1}_{m+k}) * C^{a}_{n}$.

In problem №5, the autonomous trajectory of the king contains 5 moves (a=5), in Catalan's scheme C(6, 5, 0) k+m=6+5=11 moves and one strict move (e5) (r = 1).

The number of moves in the solution is n = a+m+k+r = 17.

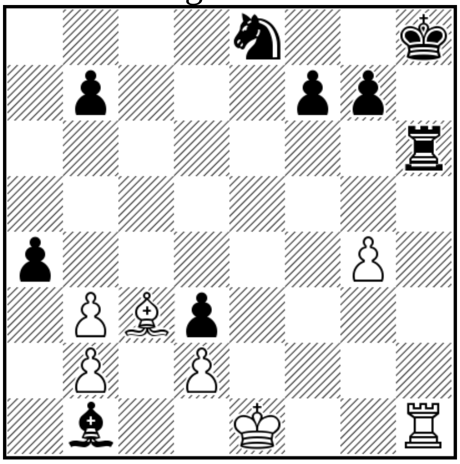
The number of solutions to the problem can be found by the formula:

$N = C^{a}_{n} * S = C^{a}_{n} * (C^{m}_{m+k} - C^{m-q-1}_{m+k}) = C^{5}_{17} * (C^{5}_{11} - C^{4}_{11}) = 6188 * (462 - 330) = 6188 * 132 = 816816.$

Content of the problem:

1. The number of solutions to the problem **N = 816816 = 816*1001. Millennium.**
2. Miniature without captures. Two underpromotions into bishops, excelsior.
3. Model epaulette mate.

№6. Eduard Eilazyan
ChessblogYG 26.12.20



Ser.h=19–N? 7+9
How many different solutions?
4n2k/1p3pp1/7r/8/p5P1/1PBp4/1P1P4/1b2K2R

Publication:
ChessblogYG
http://9148.od.ua/stati-i-zametki/
Э.Эйлазян «Обратная задача в шахматной композиции» Часть 3. №25.
26.12.20.

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31.12.20.

Chess-mathematical problem on series-helpstalemates.
Find the (basic) solution and the number of solutions N.

Solution №6.
1.Bc2 2.Bd1 3.Bf3 4.Be4 5.Bh7 6.Bg8 7.a3 8.axb2 9.b1B 10.Bc2 11.Bd1 12.Bf3 13.Be4 14.Beh7 15.Rf6 16.g5 17.Ng7 18.b5 19.b4 Bxf6 stalemat.

In problem №6, the autonomous trajectory of the b-pawn contains 2 moves (a = 2).
In Catalan's scheme, C(k, m, q) = C(6,8,2) (k=6, m=8 and q=2) k+m= 6+8=14 moves.
Three strict moves: Rf6, g5, Ng7 (r = 3).
The number of moves of the solution n = a+m+k+r = 2+14+3 = 19.
The number of solutions to the problem is found by the formula $N = S*C^2_{19}$.
$S = C^8_{14} – C^5_{14} = 3003 – 2002 = 1001$; $C^2_{19} = 171$; N = 1001*171 = 171171.
N = 171171 – Millennium-palindrome.

Content of the problem:
1. The number of solutions to the problem N = 171171 is a millennium-palindrome.
2. Model stalemate with two pinned pieces.
3.Underpromotion; three active blockings.

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January, 2022